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Automatic Noise Temperature Measurement Through Frequency Variation

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Abstract —**The dependence of two-port noise temperature on the source reflection factor does not lend itself to easy automated measurement. This paper shows that a noise analysis performed over a small frequency interval centered about the frequency of interest and with a source circuit having fast phase variations leads to a straightforward solution of the problem. The conditions for applying the procedure are broad enough to enable measuring most components like transistors and amplifiers over the entire microwave range. An example of practical implementation is presented.**

I. Introduction

The determination of the dependence of the noise temperature on the source reflection factor requires measurements with various source impedances. These are obtained manually by means of stubs, line stretchers, and the like.

Such devices are cumbersome to operate automatically, although a mechanical tuner driven by stepper motors has been proposed [1]. Another approach uses varactor-controlled tuners [2]. Due to the small capacitance variations that can be achieved through microwave varactors, this kind of tuner operates only over a limited frequency range.

Moreover, both tuners exhibit unpredictable losses, which have to be taken into account for accurate noise measurements. Accordingly, they must be determined for every setting at every frequency.

Thus, the challenge was: could one imagine a set of electrically controlled impedances or reflection factors sufficient to determine the noise temperature and meeting the extra constraint of easily accountable losses?

II. An Automated Procedure for Determining the Noise Temperature

A guide to this choice is [3], where it is shown that the dependence of the noise temperature T_n on the source reflection factor $S = |S| \cdot e^{j\varphi_s}$ can be expressed as

$$T_n = \frac{T_a + |S|^2 T_b + 2T_c |S| \cos(\varphi_s + \varphi_c)}{1 - |S|^2}. \quad (1)$$

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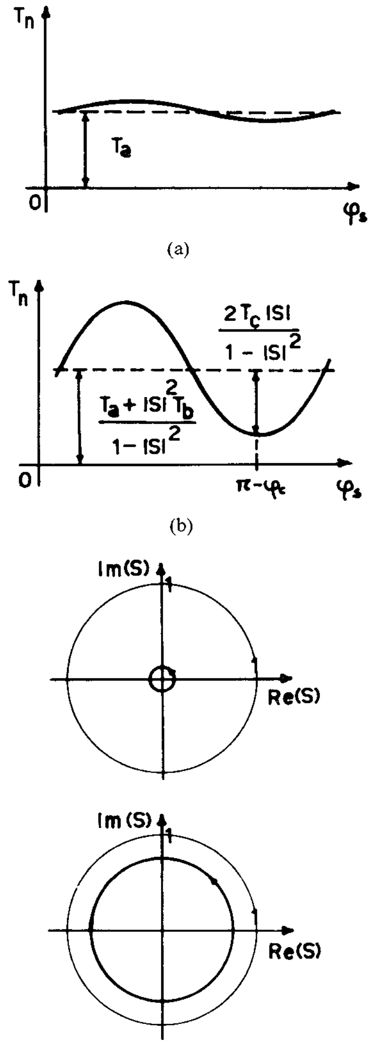


Fig. 1. The theoretical dependence of the noise temperature on the source reflection factor with (a) small $|S|$ and (b) significant $|S|$. (c) Shows the corresponding S-loci

The parameters T_a, T_b, T_c, and φ_c are related to the noise waves model for the two-port, (see [3] and [4]), and are determined experimentally in two steps.

a) *Measurements with a matched source*: With an ideal source ($S = 0$) the noise temperature would be T_a. For any real source having small mismatch $|S|$, (1) becomes

$$T_n \simeq T_a + 2T_c |S| \cos(\varphi_s + \varphi_c). \quad (2)$$

This relation is plotted as a function of φ_s in Fig. 1(a); it yields T_a as the mean value of $T_n(\varphi_s)$.

b) *Measurements with a significant source reflection factor*: The general case (1) is plotted in Fig. 1(b), where it can be seen that: the mean value of the curve gives T_b if $|S|$ and T_a are known; the amplitude of the sine wave is related to T_c; and the position of the curve defines φ_c. The S variations involved appear in Fig. 1(c). They were formerly achieved by a line stretcher inserted between the source and the DUT.

Now, automation clearly requires moving on the reflection factor loci electrically. To this end, use is made of two factors. a) Phase variations can be obtained at the output of a long section of line by slightly changing the frequency of analysis, say by 1

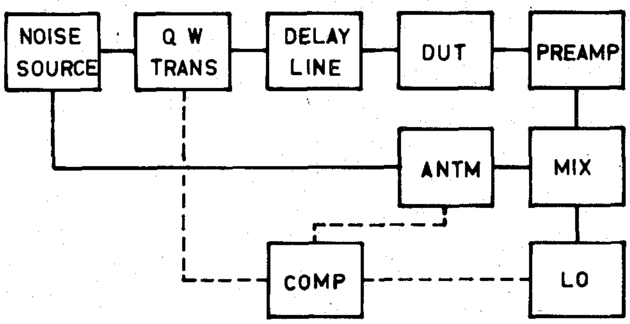
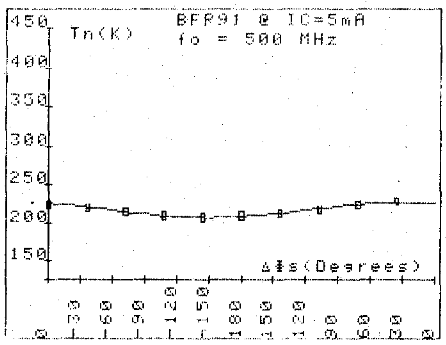
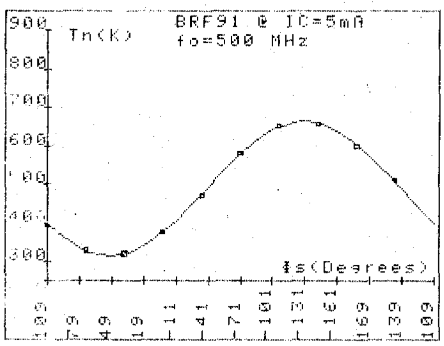


Fig. 2. Block diagram of the experimental setup.



(a)



(b)

Fig. 3. The experimental noise-temperature variation as a function of φ_s with (a) the quasi-matched noise source and (b) the quarter-wave transformer inserted.

percent. b) Over this kind of interval, the noise parameters of most devices or systems present very small changes,[1] whose first-order terms largely cancel out when averaging (see Appendix).

III. An Example of Implementation

The method was applied to a BFR 91 bipolar transistor around $f_0 = 500$ MHz. A block-diagram of the setup is given in Fig. 2.

The noise source is a conventional solid-state device. It is followed by an insertable quarter-wave transformer that produces the nonzero reflection factor needed in step b) above. Next is a 60-nS delay line that produces a 360° shift for φ_s over a $\Delta f = 8.3$-MHz interval. The noise at the output of the DUT is amplified and filtered for image frequency rejection. Consecutively, it is down converted to the 30-MHz IF of the automatic noise-temperature meter (ANTM) that also switches the noise source on and off. Measurement bandwidth is reduced to 400 KHz for adequate resolution within Δf.

[1]An exception being systems with a sharp resonance at the frequency of interest.

TABLE I
The Noise Parameters Derived from the Automatic Procedure

T_a(K)	T_b (K)	T_c (K)	φ_c(DEG)
217	150	77	130

*The D.U.T. is a BFR 91 bipolar transistor biased at $I_C = 5$ mA, $V_{CE} = 5$ V. Center frequency is 500 MHZ.

TABLE II
The Noise Parameters Derived from Manual Measurements at 495, 500, 505 MHz for the Same Device as Before

f(MHz)	T_a (K)	T_b(K)	T_c (K)	φ_c(DEG)
495	225	144	77	133
500	213	144	71	132
505	222	144	74	132

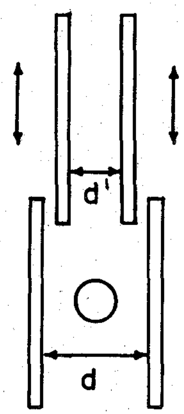


Fig. 4. Schematic view of the insertable quarter-wave transformer. Distance d corresponds to a 50-Ω characteristic impedance, d' to about 16 Ω.

The setup is controlled by a computer whose functions are: a) to insert or not the quarter-wave transformer; b) to change the frequency of analysis by action on the local oscillator; and c) to take readings from the ANTM and compute the noise parameters T_a, T_b, T_c, and φ_c.

The experimental wave forms obtained with a) the quasi-matched noise source and b) the quarter-wave transformer inserted ($|S| \simeq 0.653$) are shown in Fig. 3. The noise parameters resulting from them are listed in Table I.

To verify their accuracy and the assumption of almost constant parameters over Δf, manual measurements were performed at the three frequencies 495, 500, and 505 MHz. The results are shown in Table II.

As the differences between these two sets of results certainly lie within the experimental uncertainties, the two methods prove consistent and equivalent.

IV. About the Design of the Input Network

As was mentioned, a switchable quarter-wave transformer was used to create the significant source reflection factor needed. It is made of a 50-Ω slab line section, whose characteristic impedance can be modified by inserting two mobile plates (see Fig. 4). Actuation is by a coil magnet.

This solution proved to work quite well, as the quarter-wave transformer exhibits minimal $|S|$ variations over the interval Δf. However, it only allows operation around a single frequency. For broad-band operation, it could advantageously be replaced by an unmatched attenuator.

The frequency interval Δf can be reduced if the line length is increased. Doing this, however, increases the losses, which in turn can reduce the reflection factor as seen by the DUT below a suitable value. A tradeoff must accordingly be accepted.

Finally, accurate noise measurements demand that the excess noise ratio of the source be carefully corrected for losses. These concentrate essentially in the delay line ($\simeq 0.5$ dB) and can be measured with the highest accuracy in matched conditions. It is then a simple matter to deduce them by computation for the unmatched case.

V. Conclusion

The results presented show that a set of source impedances enabling an easy determination of the noise temperature can be achieved through small frequency changes and the use of an appropriate input network, rather than by tuning at a single frequency. Advantage was taken of this feature to devise and experiment a setup for fully automated noise measurements.

The simple nature of the required hardware allows one to forecast possible applications throughout the microwave range.

Appendix

Generally the noise parameters T_a, T_b, T_c, and φ_c in (1) are frequency dependent. Let us show that this dependence only biases the results to a negligible extent. To this aim, the noise parameters are developed into their first order expansions around f_0

$$T_a(f) \simeq T_{a_0} + \left.\frac{\partial T_a}{\partial f}\right|_{f_0} \cdot (f - f_0) \tag{3}$$

$$T_b(f) \simeq T_{b_0} + \left.\frac{\partial T_b}{\partial f}\right|_{f_0} \cdot (f - f_0) \tag{4}$$

$$T_c(f) \simeq T_{c_0} + \left.\frac{\partial T_c}{\partial f}\right|_{f_0} \cdot (f - f_0) \tag{5}$$

$$\varphi_c(f) \simeq \varphi_{c_0} + \left.\frac{\partial \varphi_c}{\partial f}\right|_{f_0} \cdot (f - f_0) \tag{6}$$

and likewise for the variable φ_s

$$\varphi_s(f) \simeq \varphi_{s_0} + \left.\frac{\partial \varphi_s}{\partial f}\right|_{f_0} \cdot (f - f_0). \tag{7}$$

Putting $\nu = f - f_0$ (1) re-writes

$$T_n(\nu) = (a_0 + b_0) + (a_1 + b_1)\cdot \nu + (c_0 + c_1 \cdot \nu) \cdot \cos\left(p_0 + (p_1' + p_1'')\cdot \nu\right)$$

with

$$p_1' = \frac{\partial \varphi_s}{\partial f} \qquad p_1'' = \frac{\partial \varphi_c}{\partial f}$$

and $a_0, b_0, \cdots,$ obtained by identification.

As sketched in Fig. 1, the quantities that have to be evaluated are the mean value, the amplitude, and the phase of an offset sine wave. This can be done mathematically through the Fourier coefficients

$$I_1 = \frac{1}{\Delta f} \cdot \int_{-\Delta f/2}^{\Delta f/2} T_n(\nu) \cdot d\nu$$

$$I_2 = \frac{2}{\Delta f} \cdot \int_{-\Delta f/2}^{\Delta f/2} T_n(\nu) \cos(p_1' \cdot \nu) \cdot d\nu$$

$$I_3 = \frac{2}{\Delta f} \cdot \int_{-\Delta f/2}^{\Delta f/2} T_n(\nu) \sin(p_1' \cdot \nu) \cdot d\nu.$$

Indeed, if the noise parameters were constant with respect to frequency, the above expression for $T_n(\nu)$ would read

$$\begin{aligned} T_n(\nu) &= a_0 + b_0 + c_0 \cdot \cos(p_0 + p_1' \cdot \nu) \\ &= a_0 + b_0 + c_0 \cdot \cos p_0 \cdot \sin p_1' \cdot \nu - c_0 \sin p_0 \cdot \cos p_1' \cdot \nu \end{aligned}$$

and the values of the integrals I_1, I_2, and I_3 would simply be

$$I_1^0 = a_0 + b_0$$

$$I_2^0 = c_0 \cdot \cos p_0$$

$$I_3^0 = -c_0 \cdot \sin p_0.$$

With the first-order approximations (3)–(7), remembering that Δf is chosen such that $p_1' \Delta f = 2\pi$ and assuming for the moment that $|p_1''| \ll |p_1'|$, one finds

$$I_1 \simeq a_0 + b_0 - c_0 \cdot \frac{p_1''}{p_1'} \cdot \cos p_0 - c_1 \cdot \frac{1}{p_1'} \cdot \sin p_0$$

$$I_2 \simeq c_0 \cdot \left(1 + \frac{1}{2} \cdot \frac{p_1''}{p_1'}\right) \cdot \cos p_0 + \frac{c_1}{p_1'} \cdot \sin p_0$$

$$I_3 \simeq -c_0 \cdot \left(1 - \frac{1}{2} \cdot \frac{p_1''}{p_1'}\right) \cdot \sin p_0 - \frac{c_1}{p_1'} \cdot \cos p_0.$$

In order that the error remain small, integrals I_j and I_j^0 must not differ significantly. Obviously the conditions for this are

$$e_1 = \left|\frac{p_1''}{p_1'}\right| = \left|\frac{\partial \varphi_c}{\partial f} \Big/ \frac{\partial \varphi_s}{\partial f}\right| \ll 1$$

and

$$e_2 = \left|\frac{c_1}{c_0} \cdot \frac{1}{p_1'}\right| = \left|\frac{1}{T_c} \cdot \frac{\partial T_c}{\partial f} \Big/ \frac{\partial \varphi_s}{\partial f}\right| \ll 1.$$

To what degree these conditions are met in our example can be gathered from the computed values

$$\left|\frac{\partial \varphi_s}{\partial f}\right| = 0.757 \text{ rad/MHz}$$

$$\left|\frac{\partial \varphi_c}{\partial f}\right| = 0 \text{ rad/MHz}$$

$$\left|\frac{1}{T_c} \cdot \frac{\partial T_c}{\partial f}\right| = 0.4 \text{ percent/MHz}$$

from which $e_1 = 0$ and $e_2 = 0.005$.

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